Math 2050, HW 3 (due: 25 Oct)

(1) If $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are two bounded sequence, show that
$$\limsup_{n\to+\infty}(x_n+y_n) \le \limsup_{n\to+\infty} x_n + \limsup_{n\to+\infty} y_n.$$
Show that the equality is not always true by an example.

(2) If $x_1 < x_2$ are some real numbers and $x_n = \frac{1}{4}x_{n-1} + \frac{3}{4}x_{n-2}$ for $n > 2$. Show that $\{x_n\}_{n=1}^{\infty}$ is convergent and find the limit.

(3) If $r \in (0,1)$ and $|x_{n+1} - x_n| < r^n$ for all $n > 1$. Show that $\{x_n\}_{n=1}^{\infty}$ is a convergent.

(4) Suppose $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence such that x_n is an integer for any $n \in \mathbb{N}$. Show that there is N such that x_n is a constant for $n > N$.